



# Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 21 Further Pure Mathematics 1

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp        decimal places
  - sf        significant figures
  - \*        The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.  
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
  6. Ignore wrong working or incorrect statements following a correct answer.
  7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

| Question                                                                                                                                                                                   | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                                                                | $x = 0 \Rightarrow 12 - 7.5 = 4.5^{\circ}\text{C}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | B1         | 3.4  |
|                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <b>(1)</b> |      |
| <b>(b)</b>                                                                                                                                                                                 | $S = 12 - \frac{15}{2} \left( \frac{1-t^2}{1+t^2} \right) - \frac{27}{10} \left( \frac{2t}{1+t^2} \right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | M1         | 1.1b |
|                                                                                                                                                                                            | $= \frac{120(1+t^2) - 75(1-t^2)3 - 54t}{10(1+t^2)}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | M1         | 1.1b |
|                                                                                                                                                                                            | $= \frac{195t^2 - 54t + 45}{10(1+t^2)}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | A1         | 2.1  |
|                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <b>(3)</b> |      |
|                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |            |      |
| <b>(c)</b>                                                                                                                                                                                 | $\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | M1         | 3.4  |
|                                                                                                                                                                                            | $95t^2 - 54t - 55 = 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | A1         | 1.1b |
|                                                                                                                                                                                            | $t = 1.09..., \Rightarrow \frac{x}{2} = \tan^{-1} 1.09... = ... \{47.4... \text{or } 0.828...\}$ <p style="text-align: center;">or</p> $-0.528... \Rightarrow \frac{x}{2} = \tan^{-1}(-0.528) = ... \{-27.83... \text{or } -0.485...\}$ <p style="text-align: center;"><b>Alternative: Leading to a value for x</b></p> $t = 1.09.. \Rightarrow \sin x = \frac{2(1.09)}{1+(1.09)^2} \text{ or } \cos x = \frac{1-(1.09)^2}{1+(1.09)^2}$ <p style="text-align: center;">Or</p> $t = -0.528.. \Rightarrow \sin x = \frac{2(-0.528)}{1+(-0.528)^2} \text{ or } \cos x = \frac{1-(-0.528)^2}{1+(-0.528)^2}$ | dM1        | 3.4  |
|                                                                                                                                                                                            | $\frac{x}{2} = \tan^{-1}(-0.528) + 180 = 152.1... \Rightarrow x = ...$ $x = 360 + \sin^{-1} \left( \frac{2(-0.528)}{1+(-0.528)^2} \right)$ $x = 360 - \cos^{-1} \left( \frac{1-(-0.528)^2}{1+(-0.528)^2} \right)$                                                                                                                                                                                                                                                                                                                                                                                       | ddM1       | 3.1b |
|                                                                                                                                                                                            | 304 days                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | A1         | 3.2a |
|                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <b>(5)</b> |      |
|                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |            |      |
| <b>(9 marks)</b>                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |            |      |
| <b>Notes</b>                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |            |      |
| <p>(a)</p> <p>B1: Obtains the correct temperature of <math>4.5^{\circ}\text{C}</math> (must include units)</p> <p>(b)</p> <p>M1: Uses the correct t-formulae to obtain S in terms of t</p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |            |      |

M1: Correct method to obtain a common denominator,

$$A + \frac{B(1-t^2)}{1+t^2} + \frac{Ct}{1+t^2} = \frac{A(1+t^2) + B(1-t^2) + Ct}{(1+t^2)}$$

A1: Correct answer in the correct form

(c)

M1: Uses  $S = 10$  with the model and multiplies up to obtain a quadratic equation in  $t$

A1: Correct 3TQ

dM1: Solves their 3TQ in  $t$  and proceeds to obtain at least one value of  $\frac{x}{2}$  as suggested by the

model. Alternative uses  $t$  formulae for  $\sin x$  or  $\cos x$  to find a value for  $x$ , degrees or radians

ddM1: A fully correct strategy to find the required value of  $x$  from the negative root of the quadratic equation in  $t$ . They must be using degrees for this mark. This mark may be awarded even if more than one value found e.g  $x = 95.2, 304.3$

A1: Correct number of days, must be clear that this is their answer

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Scheme                                                                                                                                                       | Marks | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Steps are 25 minutes so $h = \frac{25}{60}$                                                                                                                  | B1    | 3.3  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\left(\frac{dH}{dt}\right)_0 = \frac{-4\sqrt{2}}{3(5 \times 2 - 2^2)} (= -0.314...)$                                                                        | M1    | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\Rightarrow \left(\frac{dH}{dt}\right)_0 = \frac{H_1 - H_0}{\frac{5}{12}} \Rightarrow H_1 = \frac{5}{12} \left(\frac{dH}{dt}\right)_0 + 2$                  | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $= 1.86905... \left( = \frac{108 - 5\sqrt{2}}{54} \right)$                                                                                                   | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\left(\frac{dH}{dt}\right)_1 = \frac{-4\sqrt{H_1}}{3(5 \times H_1 - H_1^2)} (= -0.311...)$                                                                  | M1    | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\Rightarrow \left(\frac{dH}{dt}\right)_1 = \frac{H_2 - H_1}{\frac{5}{12}} \Rightarrow y_2 = \frac{5}{12} \left(\frac{dH}{dt}\right)_1 + H_1 (= 1.73926...)$ | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Awrt 1.74 m                                                                                                                                                  | A1    | 3.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                              | (7)   |      |
| (7 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                              |       |      |
| Notes                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                              |       |      |
| <p>B1: Uses the given information to set up the correct value of <math>h</math> for the model</p> <p>M1: Uses <math>H = 2</math> in the given differential equation to find <math>\left(\frac{dH}{dt}\right)_0</math></p> <p>M1: Uses the approximation formula with their <math>h</math> and their <math>\left(\frac{dH}{dt}\right)_0</math> to find <math>H</math> after 35 minutes</p> <p>A1: Correct value for <math>H</math> after 35 minutes (accept the exact value or awrt 1.87)</p> <p>M1: Uses their <math>H</math> after 35 minutes in the given differential equation to find <math>\left(\frac{dH}{dt}\right)_1</math></p> <p>M1: Uses the approximation formula with their <math>h</math> and their <math>\left(\frac{dH}{dt}\right)_1</math> to find <math>H</math> after 1 hour</p> <p>A1: Correct answer. Allow awrt 1.74 m with units</p> <p><b>Note</b> if they use more than two iterations, they can score a maximum of B0M1M1A0M0M0A0</p> <p>If uses <math>h = \frac{1}{6}</math> then <math>H_1 = \frac{54 - \sqrt{2}}{27} = 1.9476</math></p> |                                                                                                                                                              |       |      |

**It may be seen in a table**

| n | $H_n$    | T              | $\left(\frac{dH}{dt}\right)_n$       | $H_{n+1}$ |
|---|----------|----------------|--------------------------------------|-----------|
| 0 | 2        | $\frac{1}{6}$  | $-\frac{2\sqrt{2}}{9} = -0.314\dots$ | 1.869...  |
| 1 | 1.869... | $\frac{7}{12}$ | - 0.311..                            | 1.739..   |
| 2 |          | 1              |                                      |           |

| Question    | Scheme                                                                                                                                                                                                                                                                                                                                                       | Marks      | AOs          |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------|
| <b>3(a)</b> | $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}, \overrightarrow{AD} = \begin{pmatrix} 1 \\ -1 \\ t-7 \end{pmatrix}$                                                                                                                                                                                                                       | M1         | 1.1b         |
|             | $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -4 \\ 1 & -1 & t-7 \end{vmatrix} = \begin{pmatrix} 2t-14-4 \\ -(t-7+4) \\ -1-2 \end{pmatrix}$                                                                                                                                              | dM1        | 1.1b         |
|             | $\begin{pmatrix} 2t-18 \\ 3-t \\ -3 \end{pmatrix}$ or $(2t-18)\mathbf{i} - (t-3)\mathbf{j} - \mathbf{k}$                                                                                                                                                                                                                                                     | A1         | 2.2a         |
|             |                                                                                                                                                                                                                                                                                                                                                              | <b>(3)</b> |              |
| <b>(b)</b>  | $\overrightarrow{AE} = \begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix} \Rightarrow V = \begin{pmatrix} 2t-18 \\ 3-t \\ -3 \end{pmatrix} \cdot \begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix} = 2t^2 - 6t - 18t + 54 - 9 + 3t - 9$<br>Or<br>$\begin{vmatrix} t-3 & -3 & 3 \\ 1 & 2 & -4 \\ 1 & -1 & t-7 \end{vmatrix}$<br>$= (t-3)[2(t-7)-4] + 3[t-7+4] + 3[-1-2]$ | M1<br>A1   | 1.1b<br>1.1b |
|             | $2t^2 - 21t + 36 = 9 \Rightarrow t = \dots$<br>or<br>$2t^2 - 21t + 36 = -9 \Rightarrow t = \dots$                                                                                                                                                                                                                                                            | dM1        | 1.1b         |
|             | $2t^2 - 21t + 36 = 9 \Rightarrow t = \dots$<br>and<br>$2t^2 - 21t + 36 = -9 \Rightarrow t = \dots$                                                                                                                                                                                                                                                           | M1         | 1.1b         |
|             | $t = 9, \frac{3}{2}$ or $t = 3, \frac{15}{2}$                                                                                                                                                                                                                                                                                                                | A1         | 1.1b         |
|             | $t = 9, \frac{3}{2}$ and $t = 3, \frac{15}{2}$                                                                                                                                                                                                                                                                                                               | A1         | 2.2a         |
|             |                                                                                                                                                                                                                                                                                                                                                              | <b>(6)</b> |              |
|             | <b>(9 marks)</b>                                                                                                                                                                                                                                                                                                                                             |            |              |

### Notes

(a)

M1: Attempts to find both vectors by subtraction. Allow either direction. If no method shown then two correct values is sufficient.

dM1: Uses the correct process of the vector product. May be implied by 2 correct components if working not shown.

A1: Correct vector from correct work, isw

(b)

M1: Attempts  $\pm \overrightarrow{AE}$  and the scalar triple product using their vector from part (a). Alternatively uses determinant approach

A1: Correct expression in any form, may be unsimplified

dM1: Sets their triple product = 9 or  $-9$  and attempts to solve 3TQ using a correct method

M1: Sets their triple product = 9 and  $-9$  and attempts to solve both 3TQ's using a correct method

A1: One correct pair of values or any 2 correct values

A1: All 4 values correct

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Scheme                                                                                                                         | Marks      | AOs         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|------------|-------------|
| 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | $\frac{x-8}{x} \leq \frac{7}{x(x-2)}$                                                                                          |            |             |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $\frac{(x-2)(x-8)-7}{x(x-2)} \leq 0$ <p>or</p> $x(x-8)(x-2)^2 - 7x(x-2) \leq 0$ <p>Or</p> $x^3(x-8)(x-2)^2 - 7x^3(x-2) \leq 0$ | M1         | 2.1         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $\frac{(x-9)(x-1)}{x(x-2)} \leq 0 \text{ or } x(x-1)(x-2)(x-9) \leq 0$ <p>Or <math>x^3(x-1)(x-2)(x-9) \leq 0</math></p>        | dM1        | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Critical values 0 and 2                                                                                                        | B1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | All 4 critical values 0, 1, 2, 9                                                                                               | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $\{x \in \mathbb{R} : 0 < x \leq 1\} \cup \{x \in \mathbb{R} : 2 < x \leq 9\}$                                                 | ddM1<br>A1 | 2.2a<br>2.5 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                | (6)        |             |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                |            |             |
| <b>Notes</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                |            |             |
| <p>M1: Gathers terms on one side and puts over a common denominator, or multiplies by <math>x^2(x-2)^2</math> or <math>x^4(x-2)^2</math> and gathers terms on one side</p> <p>dM1: Factorises numerator or factorises into 4 factors</p> <p>B1: Identifies the critical values 0 and 2, may be seen in an inequality</p> <p>A1: All 4 correct critical values, must have scored dM1</p> <p>ddM1: Deduces 2 “inside” inequalities are required with critical values in ascending order as shown</p> <p>A1: Exactly 2 correct intervals using correct notation minimum <math>0 &lt; x \leq 1 \cup 2 &lt; x \leq 9</math></p> |                                                                                                                                |            |             |

| Question | Scheme                                                                                                                                                                                                                                                                             | Marks | AOs  |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 5(a)     | $\frac{dy}{dx} = \frac{8}{8t} = \frac{1}{t}$ <p>or</p> $y = 4\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{x}} = \frac{1}{t}$ <p>or</p> $2y \frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y} = \frac{1}{t}$ <p>Or</p> $\frac{dy}{dx} = \frac{2a}{y} = \frac{1}{t}$ | B1    | 1.1b |
|          | $y - 8t = \frac{1}{t}(x - 4t^2)$ <p>Or</p> $y = \frac{1}{t}x + c \Rightarrow 8t = \frac{1}{t} \times 4t^2 \Rightarrow c = \dots \{4t\}$                                                                                                                                            | M1    | 1.1b |
|          | $ty - 8t^2 = x - 4t^2 \Rightarrow yt - x = 4t^2 *$ <p>Or</p> $y = \frac{1}{t}x + 4t \Rightarrow yt - x = 4t^2 *$                                                                                                                                                                   | A1*   | 2.1  |
|          |                                                                                                                                                                                                                                                                                    | (3)   |      |
|          |                                                                                                                                                                                                                                                                                    |       |      |
| (b)      | $y = -tx$                                                                                                                                                                                                                                                                          | B1    | 2.2a |
|          | $yt - x = 4t^2, y = -tx \Rightarrow -t^2x - x = 4t^2 \Rightarrow x = \dots$ <p>or</p> $yt - x = 4t^2, y = -tx \Rightarrow yt + \frac{y}{t} = 4t^2 \Rightarrow y = \dots$                                                                                                           | M1    | 2.1  |
|          | $\left( \frac{-4t^2}{t^2+1}, \frac{4t^3}{t^2+1} \right)$ or seen as $x = \dots y = \dots$                                                                                                                                                                                          | A1    | 1.1b |
|          |                                                                                                                                                                                                                                                                                    | (3)   |      |
| (c)      | $\frac{y}{x} = -t \Rightarrow x = \frac{-4\frac{y^2}{x^2}}{\frac{y^2}{x^2}+1} \text{ or } y = \frac{-4\frac{y^3}{x^3}}{\frac{y^2}{x^2}+1}$                                                                                                                                         | M1    | 1.1b |
|          | $\Rightarrow y^2 = \dots$                                                                                                                                                                                                                                                          | dM1   | 1.1b |
|          | $y^2 = \frac{-x^3}{x+4}$                                                                                                                                                                                                                                                           | A1    | 1.1b |
|          |                                                                                                                                                                                                                                                                                    | (3)   |      |

|  |                                                                                                                                                                                                                                                                                                                                                                                                          |     |      |
|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|------|
|  | <p style="text-align: center;"><b>Alternative 1</b></p> $y^2 = \left( \frac{4t^3}{1+t^2} \right)^2 = \frac{16t^6}{(1+t^2)^2} \quad x^3 = \left( \frac{-4t^2}{1+t^2} \right)^3 = \frac{-64t^6}{(1+t^2)^3}$ <p style="text-align: center;">Leading to</p> $y^2 = -\frac{1}{4}x^3(1+t^2) \text{ or } \frac{y^2}{x^3} = -\frac{1}{4}(1+t^2)$                                                                 | M1  | 1.1b |
|  | $x = -\frac{4t^2}{(1+t^2)} \Rightarrow t^2 = -\frac{x}{x+4} \Rightarrow t^2 + 1 = \frac{4}{x+4}$ $y^2 = -\frac{1}{4}x^3 \left( \frac{4}{x+4} \right)$ <p style="text-align: center;">Or</p> $\frac{y^2}{x^3} = \frac{1+t^2}{-4} = \frac{A}{x+B} = \frac{A}{\left( \frac{-4t^2}{1+t^2} \right) + B} = \frac{A(1+t^2)}{-4t^2 + B(1+t^2)}$ <p style="text-align: center;">Leading to values for A and B</p> | dM1 | 1.1b |
|  | $y^2 = \frac{-x^3}{x+4}$                                                                                                                                                                                                                                                                                                                                                                                 | A1  | 1.1b |
|  |                                                                                                                                                                                                                                                                                                                                                                                                          | (3) |      |
|  | <p style="text-align: center;"><b>Alternative 2</b></p> $y = -tx \Rightarrow y^2 = t^2 x^2$ $x = -\frac{4t^2}{t^2+1} \Rightarrow xt^2 + x = -4t^2 \Rightarrow t^2 = -\frac{x}{4+x}$                                                                                                                                                                                                                      | M1  | 1.1b |
|  | $y^2 = \left( -\frac{x}{4+x} \right) x^2$                                                                                                                                                                                                                                                                                                                                                                | dM1 | 1.1b |
|  | $y^2 = \frac{-x^3}{x+4}$                                                                                                                                                                                                                                                                                                                                                                                 | A1  | 1.1b |
|  |                                                                                                                                                                                                                                                                                                                                                                                                          | (3) |      |
|  | <p style="text-align: center;"><b>Alternative 3</b></p> $y^2 = \left( \frac{4t^3}{1+t^2} \right)^2 = \frac{16t^6}{(1+t^2)^2} = \frac{A \left( \frac{-4t^2}{1+t^2} \right)^3}{\left( \frac{-4t}{1+t^2} \right) + B}$ $\frac{A(-64t^6)}{(1+t^2)^3} \div \frac{-4t^2 + B(1+t^2)}{(1+t^2)} = \frac{A(-64t^6)}{(1+t^2)^2(-4t^2 + B + Bt^2)}$ <p style="text-align: center;">Leading to <math>B = 4</math></p> | M1  | 1.1b |
|  |                                                                                                                                                                                                                                                                                                                                                                                                          |     |      |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                |     |      |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|-----|------|
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{A(-64t^6)}{4(1+t^2)^2} = \frac{16t^6}{(1+t^2)^2} \Rightarrow A = \dots \{-1\}$          | dM1 | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Correct values for A and B and draws the conclusion that therefore<br>$y^2 = \frac{-x^3}{x+4}$ | A1  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                | (3) |      |
| <b>(9 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                |     |      |
| <b>Notes</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                |     |      |
| <p>(a)<br/> B1: Obtains the correct tangent gradient<br/> M1: Correct strategy for the equation of the tangent<br/> A1*: Correct proof with no errors</p> <p>(b)<br/> M1: Deduces the correct equation of <math>l</math><br/> M1: Solves simultaneously their equation of the normal (using a changed gradient) and the answer to (a) to find <math>x</math> or <math>y</math><br/> A1: Correct simplified coordinates</p> <p>(b)<br/> M1: Eliminates <math>t</math> to obtain a Cartesian equation<br/> dM1: Rearranges to obtain the form <math>y^2 = f(x)</math><br/> A1: Correct equation</p> <p><b>Alternative 1</b><br/> M1: Finds their <math>y^2</math> and their <math>x^3</math> substitutes into the given equation and simplifies<br/> dM1: Uses their <math>x</math> to find an expression in terms of <math>x</math> for <math>1+t^2</math> or compares the simplified <math>\frac{y^2}{x^3}</math> with <math>\frac{A}{x+B}</math> uses correct algebra to find as a single fraction and find values for A and B<br/> A1: Achieves the correct equation</p> <p><b>Alternative 2</b><br/> M1: Finds <math>y^2</math> and uses the <math>x</math> coordinates to find <math>t^2</math> in terms of <math>x</math><br/> dM1: Substitutes their <math>t^2</math> into <math>y^2</math> to find in terms of <math>x</math> only</p> <p><b>Alternative 3</b><br/> M1: Substitutes their <math>y^2</math> and their <math>x</math> into given equation. Uses correct algebra to simplify to a single fraction to deduce a value for B.<br/> dM1: Uses their value of B and equates to find a value for A<br/> A1: Correct value for A and B and draws the conclusion therefore <math>y^2 = \frac{-x^3}{x+4}</math></p> <p><b>Note: There may be other methods, please send to review if you are unsure</b></p> |                                                                                                |     |      |

